Calculus Application Problem **Name** ____________________

The Ball Goes Up, the Ball Comes Down

Introduction: The objective of this activity is to collect data from pushing a ball up a ramp and, with a motion tector placed at the top of the ramp, record the distance vs. time data of the ball as it moves up and then back wn the ramp. We will then analyze the data by finding a model for the distance vs. time data, and a model for the velocity vs. time data.

1. Complete the chart below from the data that was collected from the experiment.

Time (sec)	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2
Distance (feet)									

2. Create a scatterplot of the data on your calculator (**Time** in **L1** and **Distance** in **L2**).
(Note: Instructions to create a scatterplot are located on D2L in the Miscellaneous Worksheets section.)

3. We want to find a function that will model the data presented. It appears that the data is "parabolic" and, therefore, a **quadratic function** should be used. So we will attempt to fit our data with a quadratic function in its **vertex form**:

$$y = a(x-h)^2 + k$$

Recall, from algebra, that the values of **h** and **k** represent the vertex (**h** , **k**) of the parabola. By tracing on the atterplot, you should be able to obtain values for **h** and **k**. (Note: You may need to "approximate" the values of **h** and **k** because the vertex might be located between two data points, not exactly on a data point.

It appears that **h** = ________ and **k** = ________

The values of **h** and **k** mean something with respect to the activity! Explain what the values of **h** and **k** represent with respect to the data collected. Be specific!

h represents ____________________

k represents ____________________

4. We can find the value of **a** for our equation using algebra. Select a point (that is not close to the vertex) from the chart, say (2.4 , ______). Substitute this point for **x** and **y** into the quadratic equation, along with the values of **h** and **k**. The only unknown is **a**, so solve the equation for **a**. Round **a** to the nearest hundredth. Show your work below.

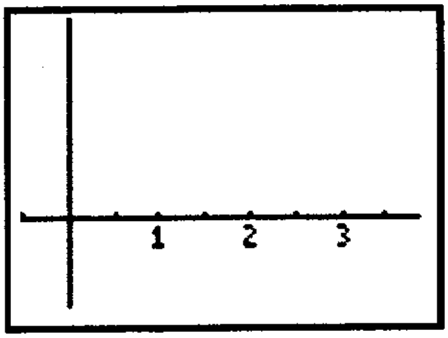
a = __________

ecord your final function. y = ____________________

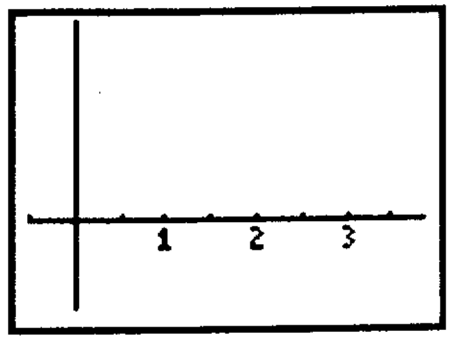
Enter this equation into **Y1** of your calculator and graph it to see how it fits the data. If it doesn't fit well, find your mistake!

5. Sketch a graph of the distance vs. time function below that you found in part #4. Then beside it, sketch a graph of the derivative of the function, which will show a graph of the velocity vs. time of the ball, since a velocity function is the derivative of a distance (position) function.

Distance



Velocity



(Note: We will come back to the velocity graph later.)

6. It is also possible to express any **quadratic function** in the **general form**:

$$y = ax^2 + bx + c$$

To determine the values of **b** and **c** (since **a** is identical to the **a** value previously found), expand your equation in part #4 and collect like terms. Round all values to the nearest hundredth. Show this work below.

y = ______________________________

7. To check your work, enter this general form of the quadratic function into **Y2** of your calculator, and see if it fits the data as well as your function in **Y1**. If not, find your mistake!

8. As another check, perform a **quadratic regression** on your data which will allow your calculator to find the "best-fitting" quadratic function (in general form) through the set of data. (This can be done by pressing the STAT key, go to CALC, and select 5:QuadReg.) Again, round all values to the nearest hundredth and record this equation below.

y = ______________________________

Compare these values of **a**, **b**, and **c** to the values found in part #6 above. (They should be close!)

Clear the functions in **Y1** and **Y2**, and enter this function in **Y1** of your calculator. Again, round values to the nearest hundredth.

For the remainder of this problem, use the **regression equation** (now in **Y1**) found in part #8 above. (You may want to turn off the scatterplot.)

9. Find the **average rate of change** of the ball over the time interval [1.6 , 2.8]. (Remember, you are using the regression equation, not the data from the chart. And also remember that there is an easy way of doing this with your calculator because your function is stored in **Y1**!)

Average rate of change from $t = 1.6$ to $t = 2.8$ is approximately ______________ ft/sec.

10. We want to find the **instantaneous rate of change** (instantaneous velocity) of the ball at the time $t = 2.0$. From our discussion in class, we know that we can estimate this value by calculating the average rate of change (average velocity) of the position of the ball over a "small time interval" that includes $t = 2.0$. Estimate the instantaneous velocity of the ball at $t = 2.0$ by calculating tha average velocity from $t = 2.0$ to $t = 2.001$. Round your answer to 2 decimal places.

Instantaneous rate of change at $t = 2.0$ is __________

11. By editing the expression that we used in #10 above we can easily calculate the instantaneous rate of change of the ball at any time t. Complete the table below that shows the **velocity** (instantaneous rate of change) of the ball at the times indicated. (Round answers to two decimal places.)

Time (sec)	0.4	1.2	2.0	2.8
Velocity (ft/sec)				

12. Create a scatterplot of this data. (**Time** in **L1** and **Velocity** in **L2.** You can **Delete** the "time" and "distance" data that are in **L1** and **L2**)

13. Since the scatterplot of the velocity vs. time data appears to be linear, choose two points from the chart above and find a linear function ($v(t) = m \cdot t + b$) that models the velocity of the ball at any time t. Show your work below, and write the function that models the data. (Round the values of m and b to two decimal places.)

$v(t) =$ ______________________

Sketch the graph of your v(t) function. Does it match the graph of your "derivative function" back in part #5 of this worksheet?

14. Use your answer to #13 above and find:

a. What was the velocity of the ball when we began collecting data? ______________

b.. Where can we find this value in the position/distance function?

____

c. Use the velocity function to determine the time that the ball reached its maximum height?

$t =$ ______________

d. Where did we see this value before?

____